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Proton Decay in the Supersymmetric Grand Unified Models

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Abstract

In this article we review proton decay in the supersymmetric grand unified models.

1 Introduction

The grand unified model (GUT) of the strong and electroweak interactions is motivated not only from an esthetic point of view, but also due to the problem of the electric charge quantization in the standard model (SM) [1]. The electric charge quantization is experimentally proved up to 10^{-21} [2]. Also, the unification of the gauge coupling constants, which is one of the GUT predictions, is shown to be valid in the supersymmetric extension of the GUT (SUSY GUT), and it is found that the GUT scale (M_{GUT}) is about 10^{16} GeV [3].

The GUT has two aspects. One is the unification of the SM gauge groups to a simple group, such as SU(5) or SO(10). The charge quantization and the gauge coupling constant unification come from the unification of gauge groups. The other is unification of matters. In the SU(5) GUT, quarks and leptons are embedded in an economical and elegant way as

$$\begin{aligned}\psi(\mathbf{10}) &= \begin{pmatrix} Q & u^c & e^c \end{pmatrix}, \\ \phi(\mathbf{5}^*) &= \begin{pmatrix} d^c & L \end{pmatrix},\end{aligned}$$

where $Q \equiv (u, d)$ and $L \equiv (\nu, e)$. Here, we adopt the left-handed basis for fermions and omit the chirality indices of fermions as far as it does not lead to confusion in this article. In the SO(10) GUT the right-handed neutrinos are introduced and the embedding becomes simpler as

$$\varphi(\mathbf{16}) = \begin{pmatrix} u^c & d^c & e^c & \nu^c \\ u & d & e & \nu \end{pmatrix}.$$

The atmospheric neutrino observation by the Super-Kamiokande experiment [4] suggests that the tau neutrino has a finite mass ($m_{\nu_\tau} \sim 10^{-(1-2)}\text{eV}$). If the tiny neutrino mass comes from the see-saw mechanism [5], the right-handed tau neutrino mass is expected to be $(10^2\text{GeV})^2/10^{-(1-2)}\text{eV} \sim 10^{15}\text{GeV}$. It is close to the GUT scale and supports the SO(10) SUSY GUT.

The unification of matters leads to violation of the global symmetries in the SM, such as baryon, lepton, and lepton flavor numbers, and then, proton decay and lepton-flavor violating processes are predicted. In this article we will concentrate on

the proton decay in the SUSY SU(5) GUT. The proton decay is the direct prediction for the model, and the search is the most important for confirmation of the GUT. For the lepton-flavor violating processes in the SUSY GUT, see Refs. [6].

There are two sources to induce the proton decay in the SUSY SU(5) GUT. One is the X boson, accompanied with the unification of gauge groups. The proton lifetime predicted by the X boson exchange is proportional to M_X^4 with M_X the X boson mass, since the X boson exchange induces the dimension-six operators. The dominant decay mode is $p \rightarrow \pi^0 e^+$. While this prediction is almost model-independent, the lifetime is sensitive to M_X . If $M_X \sim 10^{16}\text{GeV}$, the lifetime is $10^{(34-36)}$ years, which is beyond the current experimental reach.

The second one is the colored Higgs, which is introduced for doublet Higgs in the SUSY SM to be embedded into the SU(5) multiplets. In the minimal SUSY SU(5) GUT, the colored Higgs predicts much shorter proton lifetime. The colored Higgs exchange leads to the baryon-number violating dimension-five operators, and then the proton lifetime is proportional to $M_{H_C}^2$ with M_{H_C} the colored Higgs mass [7, 8, 9]. While it is suppressed by the small Yukawa coupling of quarks and leptons, the minimal SUSY SU(5) GUT is excluded from the negative search of the proton decay [10].

This paper is organized as follows. In the next section, we show the status of the minimal SUSY SU(5) GUT from a point of view of the proton decay induced by the colored Higgs, and discuss the reason why the proton decay is not still discovered if the SUSY SU(5) GUT is valid. In Section 3 we show the proton decay rate induced by the X boson, and discuss the model-dependence of the prediction. Section 4 is devoted to conclusion.

2 Proton decay induced by the colored Higgs exchange

In this section we show the status of the minimal SUSY SU(5) GUT from a point of view of the proton decay induced by the colored Higgs, and discuss the reason why the proton decay is not discovered if the SUSY SU(5) GUT is realistic.

First, let us introduce the minimal SUSY SU(5) GUT. In this model the doublet Higgs H_f and \overline{H}_f in the SUSY SM are embedded into the $\mathbf{5}$ and $\mathbf{5}^*$ dimensional multiplets as

$$H = (H_C, H_f), \quad \overline{H} = (\overline{H}_C, \overline{H}_f), \quad (1)$$

with the colored Higgs H_C and \overline{H}_C . The superpotential of the Yukawa coupling for the doublet and colored Higgs is given as

$$\begin{aligned} W_Y = & h^i (Q_i \cdot H_f) u_i^C + V_{ij}^* f^j (Q_i \cdot \overline{H}_f) d_j^C + f^i e_i^C (L_i \cdot \overline{H}_f) \\ & + \frac{1}{2} h^i e^{i\varphi_i} (Q_i \cdot Q_i) H_C + V_{ij}^* f^j (Q_i \cdot L_j) \overline{H}_C \\ & + h^i V_{ij} u_i^C e_j^C H_C + e^{-i\varphi_i} V_{ij}^* f^j u_i^C d_j^C \overline{H}_C, \end{aligned} \quad (2)$$

where the indices i and $j (= 1 - 3)$ are for the generations. V_{ij} is the Kobayashi-Maskawa matrix element at the GUT scale, and φ_i are for additional degrees of freedom in the Yukawa coupling constants in the minimal SUSY SU(5) GUT. (See Ref. [9] for notation and conversion.) Here, we use the same letters for the superfields as the component fields. The first-three terms correspond to the Yukawa coupling in the SUSY SM. As well-known, the charged leptons and the down-type quarks have common Yukawa coupling constants.

In the minimal SUSY SU(5) GUT H_C and \overline{H}_C have a common mass term by themselves, and then, the colored Higgs exchange gives following dimension-five operators,

$$W_5 = \frac{1}{2M_{H_C}} h^i e^{i\varphi_i} V_{kl}^* f^l (Q_i \cdot Q_i) (Q_k \cdot L_l) + \frac{1}{M_{H_C}} h^i V_{ij} e^{-i\varphi_k} V_{kl}^* f^l u_i^C e_j^C u_k^C d_l^C. \quad (3)$$

Since squarks or sleptons are on the external lines of these operators, Higgsino or gaugino is exchanged between them so that proton can decay (Fig. (1)). The contractions of the indices for the gauge symmetries in Eq. (3) are understood as

$$(Q_i \cdot Q_i) (Q_k \cdot L_l) = \varepsilon_{\alpha\beta\gamma} (u_i^\alpha d_i^\beta - d_i^\alpha u_i^\beta) (u_k^\gamma e_l - d_k^\gamma \nu_l), \quad (4)$$

$$u_i^C e_j^C u_k^C d_l^C = \varepsilon^{\alpha\beta\gamma} u_{i\alpha}^C e_j^C u_{k\beta}^C d_{l\gamma}^C, \quad (5)$$

with α, β , and γ being color indices. Note that the total antisymmetry in the color indices requires that the operators are flavor non-diagonal ($i \neq k$). Therefore the

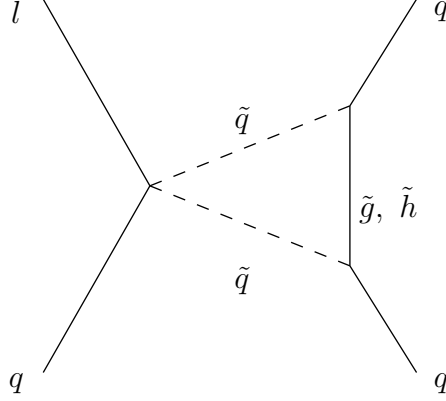


Figure 1: Higgsino and gaugino dressing to the baryon-number violating dimension-five operators.

dominant decay modes tend to have strangeness. In the minimal SUSY SU(5) GUT the dominant mode is $p \rightarrow K^+ \bar{\nu}$ while $p \rightarrow K^0 \mu^+$ is not. The mode of $K^0 \mu^+$ is suppressed by $(m_u/m_c V_{ud} V_{cd})^2$.

The largest uncertainty to the decay rate predicted by the colored Higgs exchange comes from the colored Higgs mass itself in the minimal SUSY SU(5) GUT. However, the mass spectrum at the GUT scale can be constrained and the colored Higgs mass can be evaluated from the low energy parameters [11]. Due to the gauge coupling unification in the minimal SUSY SU(5) GUT we can derive the following relations from the renormalization group equations of the gauge coupling constants at one-loop level,

$$(-2\alpha_3^{-1} + 3\alpha_2^{-1} - \frac{3}{5}\alpha_Y^{-1})(m_Z) = \frac{1}{2\pi} \left\{ \frac{12}{5} \ln \frac{M_{H_C}}{m_Z} - 2 \ln \frac{m_{SUSY}}{m_Z} \right\}, \quad (6)$$

$$(-2\alpha_3^{-1} - 3\alpha_2^{-1} + 3\alpha_Y^{-1})(m_Z) = \frac{1}{2\pi} \left\{ 12 \ln \frac{M_X^2 M_\Sigma}{m_Z^3} + 8 \ln \frac{m_{SUSY}}{m_Z} \right\}. \quad (7)$$

Here, we take the SUSY particle masses in the SUSY SM to be common (m_{SUSY}). M_Σ is the mass for the SU(3)_C octet and SU(2)_L triplet components in the **24**-dimensional Higgs for SU(5) gauge symmetry breaking, which survive as the physical

degrees of freedom after the Higgs mechanism. In Ref. [12] the upperbound on M_{H_C} is derived from Eq. (6), including the various correction to it, as

$$M_{H_C} \leq 2.4 \times 10^{16} \text{GeV}. \quad (8)$$

On the other hand, the recent study for the proton decay [10] shows

$$M_{H_C} \geq 7.2 \times 10^{16} \text{GeV} \quad (9)$$

from the experimental bound on the proton lifetime $\tau(p \rightarrow K^+ \bar{\nu}) > 6.7 \times 10^{32}$ years [13], assuming the squark and slepton masses are lighter than 1TeV. Then, the minimal SUSY SU(5) GUT is excluded now.

While the minimal SUSY SU(5) GUT is excluded, the prediction of the proton decay induced by the colored Higgs exchange is itself highly model-dependent. Let us discuss the model-dependence. First, though it depends on the Yukawa coupling structure at the GUT scale, we cannot explain the mass relations between the down-type quarks and the charge leptons in the first and second generations in an SU(5) symmetric way as Eq. (2). We need to add some modification to it. One of the examples is introduction of the higher dimensional operators suppressed by the gravitational scale. If the Yukawa coupling of the colored Higgs is suppressed automatically or accidentally, the lowerbound on M_{H_C} becomes looser. In this case, the mode of $K^0 \mu^+$ may be dominant. One of the explicit models is given in Ref. [14].

Second, in some models the effective mass for the colored Higgs, which suppressed the dimension-five operators, can be much heavier than the GUT scale [15, 16, 17]. The dimension-five operators given in Eq. (3) are induced since H_C and \overline{H}_C have a common mass term. If H_C and \overline{H}_C have mass terms independent of each others by introduction of H'_C and \overline{H}'_C , the dimension-five operators are not induced.

If the Peccei-Quinn (PQ) symmetry [18] is introduced, this mechanism works [15, 16]. The PQ charges for matters and Higgs are assigned as

$$\begin{aligned} Q_{\text{PQ}}(\psi_i) &= 1, & Q_{\text{PQ}}(\phi_i) &= 1, \\ Q_{\text{PQ}}(H) &= -2, & Q_{\text{PQ}}(\overline{H}) &= -2, \\ Q_{\text{PQ}}(H') &= 2, & Q_{\text{PQ}}(\overline{H}') &= 2, \end{aligned}$$

where $H'(\equiv (H'_C, H'_f))$ and $\overline{H}'(\equiv (\overline{H}'_C, \overline{H}'_f))$. This symmetry prohibits the mass term $\sim M_{H_C} \overline{H}_C H_C$ and the dimension-five operators in Eq. (3). After the PQ symmetry is broken at $M_{\text{PQ}} \sim 10^{(10-12)}$ GeV, the dimension-five operators are generated. However, they are suppressed by $M_{\text{PQ}}/M_{\text{GUT}} \sim 10^{-(4-6)}$ and they are harmless.

In order to confirm the SUSY GUT, we need more model-independent predictions, which come from the gauge sector. One of them is the proton decay induced by the X boson. Then, we would like to discuss the model-dependence in next section.

3 Proton decay induced by the X boson exchange

In this section we show the proton decay rate induced by the X boson exchange, and discuss the model-dependence. The X boson, is $\text{SU}(3)_C$ $\mathbf{3}^*$ - and $\text{SU}(2)_L$ $\mathbf{2}$ -dimensional, and the hypercharge is $-\mathbf{5}/\mathbf{6}$. The interaction of the X boson to matter fermions is given as follows;

$$\begin{aligned} \mathcal{L} = & -\frac{1}{\sqrt{2}}g_5 V_{ij} \overline{d}_j^C (X_\mu \cdot \gamma^\mu L_i) + \frac{1}{\sqrt{2}}g_5 e^{-i\varphi_i} \overline{Q}_i X_\mu \gamma^\mu u_i^C \\ & + \frac{1}{\sqrt{2}}g_5 V_{ij} (\overline{e}_j^C \cdot X_\mu) \gamma^\mu Q_i + h.c.. \end{aligned} \quad (10)$$

This interaction leads to the following baryon-number violating operators, which contribute to $p \rightarrow \pi^0 e^+$,

$$\begin{aligned} \mathcal{L}_{eff} = & A_R \frac{g_5^2}{M_X^2} e^{i\varphi_1} \times \\ & \epsilon_{\alpha\beta\gamma} ((\overline{d}_L^C)^\alpha (u_R)^\beta (\overline{u}_R^C)^\gamma (e_L) + (1 + |V_{ud}|^2) (\overline{d}_R^C)^\alpha (u_L)^\beta (\overline{u}_L^C)^\gamma (e_R)) \end{aligned} \quad (11)$$

where A_R is the renormalization factor from the anomalous dimensions to these operators.

The renormalization factor A_R has the short-distance contribution ($A_R^{(SD)}$) and the long-distance contribution ($A_R^{(LD)}$). $A_R^{(SD)}$ at one-loop level is given as

$$A_R^{(SD)} = \left(\frac{\alpha_3(m_Z)}{\alpha_5} \right)^{\frac{4}{3b_3}} \left(\frac{\alpha_2(m_Z)}{\alpha_5} \right)^{\frac{3}{2b_2}} \quad (12)$$

where $b_3 = 9 - 2n_g$ and $b_2 = 5 - 2n_g$ with n_g the number of the generations [19]. Here, we do not include the $U(1)_Y$ contribution. It is expected to be smaller while it has not been calculated completely. If the SUSY SM is the low energy effective theory below the GUT scale, $A_R^{(SD)} = 2.1$ for $\alpha_3(m_Z) = 0.116$, $\sin^2 \theta_W = 0.2317$, and $\alpha^{-1}(m_Z) = 127.9$. The long-distance part $A_R^{(LD)}$ is

$$A_R^{(LD)} = \left(\frac{\alpha_3(m_b)}{\alpha_3(m_Z)} \right)^{\frac{6}{23}} \left(\frac{\alpha_3(\mu_{had})}{\alpha_3(m_b)} \right)^{\frac{6}{25}} = 1.2. \quad (13)$$

From the effective lagrangian (11) the proton lifetime from $p \rightarrow \pi^0 e^+$ is

$$\Gamma(p \rightarrow \pi^0 e^+) = \alpha_H^2 \frac{m_p}{64\pi f_\pi^2} (1 + D + F)^2 \left(\frac{g_5^2}{M_X^2} A_R \right)^2 (1 + (1 + |V_{ud}|^2)^2), \quad (14)$$

where m_p is the proton mass, f_π is the pion decay constant, and D and F are the chiral lagrangian parameters. The definition of α_H is

$$\alpha_H u_L(\mathbf{k}) \equiv \epsilon_{\alpha\beta\gamma} \langle 0 | (\overline{d_L^C})^\alpha (u_R)^\beta (\overline{u_R^C})^\gamma | p_{\mathbf{k}} \rangle. \quad (15)$$

α_H had one-order ambiguity in the old theoretical calculations and it led to a large uncertainty to the prediction. The latest evaluation by lattice calculation [20] gives

$$\alpha_H = -(0.015 \pm 1) \text{GeV}^3. \quad (16)$$

They adopt the naive dimensional renormalization scheme with the renormalization scale (μ_{had}) 2.3GeV. This lattice calculation is the first realistic one, in which enough a large lattice spacing and a large statistics are prepared. While this calculation still has ambiguities from the quenched approximation and the a^{-1} correction, they are expected to be at most $O(10)\%$. Finally, we get

$$\begin{aligned} 1/\Gamma(p \rightarrow \pi^0 e^+) &= 1.0 \times 10^{35} \text{years} \\ &\times \left(\frac{\alpha_H}{0.015 \text{GeV}^3} \right)^{-2} \left(\frac{\alpha_5}{1/25} \right)^{-2} \left(\frac{A_R}{2.5} \right)^{-2} \left(\frac{M_X}{10^{16} \text{GeV}} \right)^4. \end{aligned} \quad (17)$$

In Fig. (2) we show the lifetime of proton as a function of M_X . The shaded region has been excluded by Super-Kamiokande experiment. The dash-dotted line is the reach of ten years run of the Super-Kamiokande experiment. In Fig. (2) we

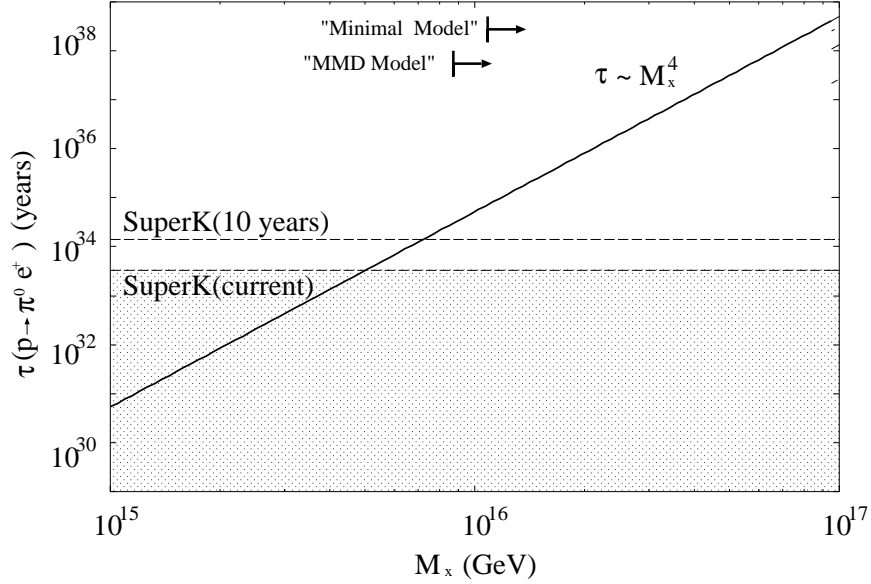


Figure 2: Lifetime of proton as a function of M_X . The shaded region has been excluded by Super-Kamiokande experiment (3.3×10^{33} years). The dash-dotted line is the expected reach of ten years run of the Super-Kamiokande experiment (1.4×10^{34} years) [22].

show the lowerbound on the X boson mass in the minimal SUSY SU(5) GUT as a reference point [9],

$$M_X \geq 1.1 \times 10^{16} \text{ GeV}. \quad (18)$$

This bound comes from the constraint from the gauge coupling unification given in Eq. (7),

$$1.3 \times 10^{16} \text{ GeV} \leq (M_X^2 M_\Sigma)^{\frac{1}{3}} \leq 3.2 \times 10^{16} \text{ GeV} \quad (19)$$

and validity of perturbation below the gravitational scale,

$$M_\Sigma/M_X = \lambda_\Sigma/2\sqrt{2}g_5 \leq 1.8. \quad (20)$$

λ_Σ the self-coupling constant of the **24**-dimensional Higgs multiplet, and if λ_Σ is much larger than g_5 , it blows up below the gravitational scale. Similarly, we can derive the lowerbound on M_X in the Modified Missing Doublet (MMD) model as

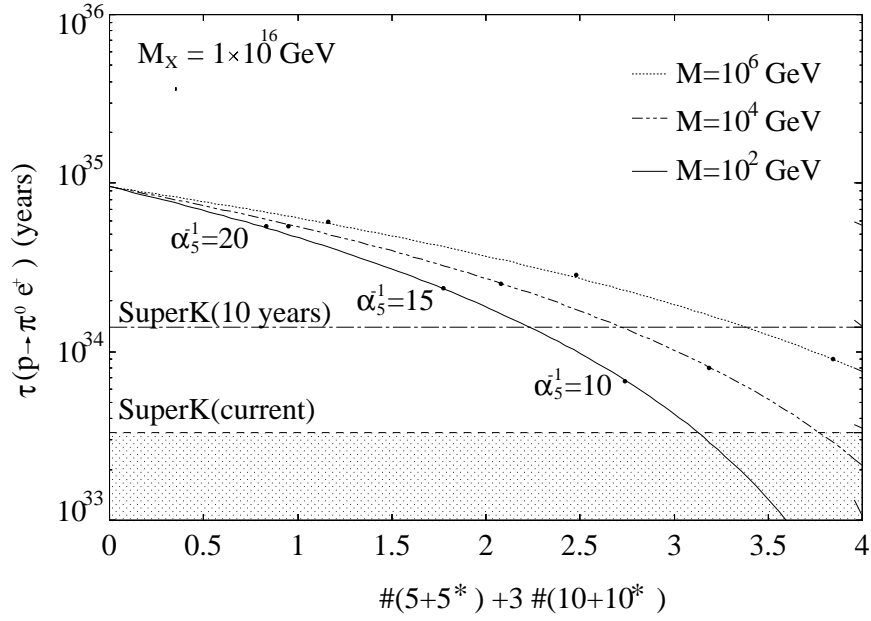


Figure 3: Lifetime of proton as a function of contribution to the beta functions of the gauge coupling constants from the extra matters. Here, we take $M_X = 1.0 \times 10^{16}$ GeV. The masses for the extra matters M are taken to be 10^2 , 10^4 , and 10^6 GeV.

$M_X \geq 8.7 \times 10^{15}$ GeV [21]. From this figure, it is found that we need further effort to confirm or reject the SUSY GUT.

We have assumed that the SUSY SM is valid below the GUT scale. However, even if extra SU(5) complete multiplets have smaller masses than the GUT scale, the success of the gauge coupling unification is not spoiled. In fact, some models, such as the gauge mediated SUSY breaking model [23], the anomalous U(1) SUSY breaking model given in Ref. [24], and the E_6 model [25], predict existence of the extra matters at lower energy. In this case the gauge coupling constant at the GUT scale becomes larger. Also, the renormalization factor A_R is enhanced as Eq. (12). Then, for fixed M_X , the proton lifetime becomes shorter.

In Fig. (3) we show the proton lifetime as a function of contribution to the beta functions of the gauge coupling constants from the extra matters. The contribution from a pair of **5** and **5*** is one, and that from a pair of **10** and **10*** is three. Here,

we fix $M_X = 1.0 \times 10^{16}$ GeV. and take the extra matter mass M to be 10^2 , 10^4 , and 10^6 GeV. From this figure, introduction of two extra generations (a pair of **5** and **5*** and a pair of **10** and **10***) with the masses 10^2 GeV is slightly disfavored.

4 Conclusion

The proton decay search is the most important for confirmation of the GUT. Now the minimal SUSY SU(5) GUT is excluded by the negative search since the dimension-five operators induced by the charged Higgs exchange predict a large proton decay rate. However, the proton decay by the colored Higgs is highly model-dependent, and it is premature to conclude that the SUSY GUT is excluded. We hope that the search will be pushed as far as possible.

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References

- [1] H. Georgi and L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
- [2] C. Caso et al, Particle Data Group, Eur. Phys. J. C3 (1998) 1.
- [3] P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817;
U. Amaldi, W.de Boer, and H.Furstenau, Phys. Lett. B260 (1991) 447.
- [4] The Super-Kamiokande Collaboration, Phys. Rev. Lett. 81 (1998) 1562.
- [5] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe*, eds. O. Sawada and A. Sugamoto (KEK, 1979) p.95;
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979).
- [6] R. Barbieri and L. J. Hall, Phys. Lett. B338 (212) 1994;
R. Barbieri, L. Hall, and A. Strumia, Nucl. Phys. B445 (219) 1995;
J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, Phys. Lett. B391 (341) 1997;

- J. Hisano, D. Nomura, Y. Okada, Y. Shimizu, and M. Tanaka, Phys. Rev. D58 (116010) 1998;
 J. Hisano, D. Nomura, and T. Yanagida, Phys. Lett. B437 (1998) 351.
- [7] N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533;
 S. Weinberg, Phys. Rev. D26 (1982) 287.
- [8] P. Nath, A. H. Chamseddine and R. Arnowitt, Phys. Rev. D32 (1985) 2348.
- [9] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B402 (1993) 46.
- [10] T. Goto and T. Nihei, Phys. Rev. D59 (1999) 115009.
- [11] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. 69 (1992) 1014.
- [12] J. Hisano, T. Moroi, K. Tobe and T. Yanagida, Mod. Phys. Lett. A10 (1995) 2267.
- [13] Y. Hayato *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. 83 (1999) 1529.
- [14] I. Gogoladze and A. Kobakhidze, Phys. Atom. Nucl. 60 (1997) 126.
- [15] J. Hisano, H. Murayama and T. Yanagida, Phys. Lett. B291 (1992) 263.
- [16] J. Hisano, T. Moroi, K. Tobe and T. Yanagida, Phys. Lett. B342 (1995) 138.
- [17] K. S. Babu and S. M. Barr, Phys. Rev. D48 (1993) 5354.
- [18] R. D. Peccei and H. R. Quinn, Phys. Rev. 38 (1977) 1440.
- [19] L. E. Ibanez and C. Munoz, Nucl. Phys. B245 (1984) 425.
- [20] S. Aoki *et al.* [JLQCD Collaboration], hep-lat/9911026.
- [21] J. Hisano, Y. Nomura and T. Yanagida, Prog. Theor. Phys. 98 (1997) 1385.
- [22] M. Shiozawa, presented at International Workshop on Next Generation Nucleon Decay and Neutrino Detector (NNN99), Stony Brook, Sep. 23-25, 1999.
- [23] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53 (1996) 2658.
- [24] J. Hisano, K. Kurosawa and Y. Nomura, Phys. Lett. B445 (1999) 316;
 hep-ph/0002286.

- [25] For a review of phenomenological aspects of the string-inspired models, see for instance, J. L. Hewett and T. G. Rizzo, Phys. Rep. 183 (1989) 193.